Nagle comments (3/28/2021) on **Implementing partisan symmetry: Problems and paradoxes**

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On p. 12 it is written “The basic idea here is extremely simple and readers can try it for themselves. Choose any four numbers from 0 to 1 whose mean is 0.7155. If one of them is at or below 0.5. (a Democratic-won seat), you will find that the median of the four scores is greater than the mean, so the mean-median score is positive.”

Consider the four numbers 0.49, 0.7, 0.7, 0.972. The mean is 0.7155 and the median is 0.7, so the median is greater than the mean, contrary to their first statement. (Their reference to mean-median does not signify “mean minus median” because it is median minus mean that has the relevant sign). This is a rather embarrassing contradiction to their invitation to the reader. However, while this reveals shoddy proofreading, this is essentially a typo. The major theme of the paper has validity, although I think the magnitude of the problem is overblown.

The major finding is that, of two maps for an unbalanced state, the one with symmetry bias favoring the majority paper may give more seats to the minority party than a map with less or no symmetry bias. That rightfully raises the issue of whether symmetry bias is valid. The paper focusses on the median minus mean (mM) and one that they call PG. PG is what I call Global Symmetry (GS) which in turn is the special case of Grofman’s 1983 method 7, and it is Grofman who they cite for their PG. They also mention PB but don’t deal with it. PB is what I call aS, which is the deviation in seats from 50% when the vote is 50%. Their contradictions also occur for PB.

Diagram

Description automatically generatedLet me show how the contradiction can arise in this figure. The Seats/Votes curve is clearly not symmetric, so PG (GS) is non-zero and seats deviation PB (aS) and mM (essentially aV) favor Republicans. In contrast, the blue line is an S(V) curve that has PB=PG=mM=0 but it would give Dems fewer seats at the most likely 0.417 D vote at the starred symbol. That is the contradiction in this paper. The responsiveness of the blue line is very high. There would be no contradiction for the green line until the most likely vote was less than 35% Dem, but then the same contradiction would again occur.

This illustrates that the contradiction regarding bias is convolved with the overall responsiveness of a plan, something that has been, of course, well known in this context. Appendix B in the recent 2020 Katz et al. paper addresses it quite nicely, although that paper doesn’t resolve the issue as the DeFord paper emphasizes. And Nagle & Ramsay dance around the issue in their Appendix C.

Let me digress to note that contradictions are known as counterexamples and they are quite devastating in rigorous mathematics. In my 2015 paper I showed a counterexample for the mM measure and one for the PB measure, but I recognized that counterexamples should be taken to be cautionary rather than absolute rejections of a measure, and so does this paper. That is why DeFord went to some effort to claim that the contradictions they report are frequent and strongly contradictory. I believe that they have exaggerated the strength of the contradictions in three ways. The first way was to give pride of presentation to a state with a small number of districts so that the inevitable small number effect can play a larger role. The second way was to push forward statewide elections that were relatively lopsided. The above figure emphasizes that symmetrical plans with smaller responsiveness are more likely to give a contradiction when the vote is more unbalanced. These two features are not disregarded by DeFord. It is more a manner of presentation that makes them appear exaggerated.

The third way that the contradictions are exaggerated is not mentioned in the DeFord paper. It is the way that one counts seats in evaluating measures of bias. While winner takes all in a district is the outcome of an election, it doesn’t properly take into account the partisan preference of a district when evaluating the partisan bias of a map. A district with partisan preference 50.00001% Dem is really only half a Dem seat from the point of view of estimating the bias of a plan. In the case of Utah, the minimum number of seats will be close to 3.5 instead of exactly 3 and less than 4. Likewise, the seat distributions in Fig. 5 will become much narrower so the minority protection at the root of the contradiction will become less strong, involving seat differences much less than the 7 seats in TX in Fig. 5. This difference then becomes relatively unimportant for guaranteeing minority representation. For unbalanced states like TN in the above figure, the responsiveness is quite small and one could argue that higher responsiveness is more important than the expected fraction of minority seats. Of course, this requires quantitative analysis. DeFord et al. have deposited their simulation data on github. It would be an appropriate project to use those data to see how much the use of fractional seats ameliorates the criticism of symmetry measures of bias.